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Inventory Continuous Model and Discrete in Economic

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Abstract

The available of continuous and discrete model, illustrate the market effect in determination of volume and market price. This effect explains the average convergen and oscilation by determination the parameter exist before. Comparing to Cobweb with discrete model result the equilibrium while Cobweb with continuous model will have equilibrium where supply and demand price is the same.

Key-words: Cobweb Model, Continuous and Discrete

1. Introduction

Two linear basic models, continous and discrete can be expanded with many ways, these do not mean that these models do not need to be aligned or perfect. A condition that associated with the economy is assumed for checking in constructing a model of course getting style that is important in a system or some aspect without complicates the analysis [7]. In some condition these models are not meant to be applied immediately, but more theoretical and built to clarify way of thinking and formal arguments, for example, in the model Cobweb. The market does not drive the price and quantity of the equilibrium value for a certain constant value. This could be a problem due to the price, while the price is never assumed in the model, so that the wrong price expectations can push market instability [1].

Therefore, in this continuous model and discrete, market influence is very important namely how other factors may affect determination of the amount and the market price. The main purpose of this model marks how the framework of a linear model, then look at the concept of time lapses and unify the discrete model and continuous models.[3]

2. Theory

2.1. Inventory On Simple Continuous Model

If market price = p; quantity of demand = D and quantity of supply = φ; then the equation of supply and demand can be written as follows:[1]

$$D = a + bp \tag{1}$$

$$\varphi = c + dp \tag{2}$$

with a, b, c and d are constants.

It is assumed that the product (goods) can be stored and inputs (demand) is not equal to the output (supply). Production inventories will increase when expenditure exceeds demand and

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will go down when demand exceeds expenditure. If it is assumed that the level of inventory (S) always be sufficient to satisfy the demand, then the valuation of inventory variables (S) can be written [4]:

$$\frac{dS}{dt} = \varphi - D \tag{3}$$

do integral at the equation (3) to obtain the value of S, *i,e*:

$$\int_0^t \frac{dS}{dt} = \int_0^t (\varphi - D)$$

$$S\big|_0^t = \int_0^t (\varphi - D)dt$$

$$S(t) = S(0) + \int_0^t (\varphi - D)dt \tag{4}$$

Furthermore, the price adjustment equation can be written:

$$\frac{dp}{dt} = -\gamma \frac{ds}{dt} \tag{5}$$

with γ are positive constants. Equation (5) are stated that the price adjustment is proportional to the supply, both accumulate and are being spent. However, because of the cost associated with stock ownership and want supplies maintenance at a certain level, it is possible that prices will fall when excess supply levels and will rise when the supply decreases [6].

2.2. Inventory On Simple Discrete Models

In general, output based on the price that is expected to be same as the market price in the previous period. With this assumption, it was found that market could become unstable at steady state [8]. Market stability is important and interesting to see if it is developed in a model that assume expected value (expectation) in price can be formed by considerations of price information from prior period amounts [5]. The goal is to expand the simple Cobweb model with assumption that manufacturers expect the general price in the market and not only related to the net price of the market before, but based on the information from the two previous periods. Furthermore, the effect of this assumption of market stability will be investigated. from the two previous periods. Furthermore, the effect of this assumption of market stability will be investigated.

3. Methodology

Adjustment of the price equation on continuous model which takes into consideration the real level of inventory irregularities of some desired or optimum level so, are:

$$\frac{dp}{dt} = -\gamma \frac{dS}{dt} + \lambda(S_o - S) \tag{6}$$

with λ is a positive constant then resulting in linear models considered it feasible to assume that the optimal inventory level *S0* depending on demand (D), so that the relationship of S_o and D stated:

$$S_0 = l\,D + m \quad \text{and} \quad \frac{dS}{dt} = \varphi - D \tag{7}$$

with l and m are positive constants [9].

Equation (1) to (7) show a complete model, with variable S, $S0$, φ, D, and p are endogenous variables that the plots are determined by the model, such as wage rates, material costs, price index and income. These variables will affect the flow of endogenous variables to the coefficients a, b, c and d. Furthermore, the above models will be resolved with the assumption that the coefficients are constant

In Cobweb models, supplies are depend on price expectations $\hat{p}_1$. Net market price is in the previous period, so it can be written:

$$\varphi_t = c + d\hat{p}_1 \tag{8}$$

and $$\hat{p}_t = p_{t-1} \tag{9}$$

with $\hat{p}_1$ is assumed related to both net price, i.e : the previous net market price and slow change price. Because of it's price expectations assume Cobweb model can be replaced as follows:[2]

$$\hat{p}_t = p_{t-1} + \rho\left(p_{t-1} - p_{t-2}\right) \tag{10}$$

with ρ is constant and is called coefficient on expectations. From equation (1) and (2) could be assumed to be fixed, then the discrete inventory model, complete is :

$$D_t = a + bp \tag{11}$$

$$\varphi_t = c + d\,[\mathrm{p}_{t-1} + \rho(p_{t-1} - p_{t-2})] \tag{12}$$

therefore $$D_t = \varphi_t\,. \tag{13}$$

4. Main Results

4.1. Continous Model Solution

Eliminate D,φ and S_0 into Equation (6) which will produce two simultaneous differential equations.

$$\frac{dp}{dt} = -\gamma\frac{dS}{dt} + \lambda\left(S_o - S\right)$$

$$s_0 = lD + m$$

with $$\frac{ds}{dt} = Q \quad D$$

$$\frac{dp}{dt} = -\gamma\left(Q - D\right) + \lambda\left(lD + m - S\right)$$

$$\frac{dp}{dt} = \quad \gamma\left(c + d_p \quad \left(a + b_p\right)\right) + \lambda\left(l\left(a + b_p\right) + m \quad S\right)$$

$$\frac{dp}{dt} = -\gamma\left(c - a\right) - \left\{\gamma\left(d - b\right) - \lambda\, lb\right\}p + \lambda\left(la + m\right) - \lambda S$$

$$\frac{dp}{dt} + \left\{\gamma\left(d \quad b\right) \quad \lambda\, lb\right\}p + \lambda S = \gamma\left(a \quad c\right) + \lambda\left(la + m\right) \tag{14}$$

Substitusi equation (14) to (6) it is obtained

$$\left\{-\gamma\left(d - b\right) + \lambda\, lb\right\}p - \lambda S + \gamma\left(a - c\right) + \lambda\left(la + m\right) = -\gamma\frac{dS}{dt} + \lambda\left(lD + m - S\right)$$

$$\frac{dS}{dt} - \left(d - b\right)p = c - a \tag{15}$$

at balance state $\frac{dp}{dt}=0$ and $\frac{dS}{dt}=0$, thus φ = D and S = S_0, and produce:

$$p^* = \frac{c-a}{d-b} \tag{16}$$

with p * is the equilibrium price.

$$S^* = lD + m$$
$$= l(a + bp^*) + m$$
$$= la + lbp^* + m$$

substitute equation (16), it is obtained:

$$S^* = la + lb\left(\frac{c-a}{d-b}\right) + m \tag{17}$$

From equation (14) and (15), can be said will reach a state of equilibrium if $p = p$ * and $S = S$ *, then the equation turns into

$$\frac{dp^*}{dt} + \{\gamma(d-b) - \lambda lb\}p^* + \lambda S^* = \gamma(a-c) + \lambda(la+m) \tag{18}$$

$$\frac{dS^*}{dt} - (d-b)p^* = c - a \tag{19}$$

reduction from the two equations (18) and (14) produce:

$$\left[\frac{d}{dt} + \alpha\right](p - p^*) + \lambda(S - S^*) = 0 \tag{20}$$

reduction from the two equations (19) and (15) produce:

$$\frac{d}{dt}(S - S^*) - \beta(p - p^*) = 0 \tag{21}$$

with

$$\alpha = \gamma(d-b) - \alpha\, lb \qquad and \qquad \beta = d - b$$

Next, eliminate $S - S^*$ from Equation (20) and (21) yields

$$S - S^* = -\frac{1}{\lambda}\left\{\frac{d}{dt} + \alpha\right\}(p - p^*) \tag{*}$$

substitute (*) into equation (21) yields

$$\frac{d}{dt}\left[-\frac{1}{\lambda}\left\{\frac{d}{dt} + \alpha\right\}(p - p^*)\right] - \beta(p - p^*) = 0 \tag{**}$$

multiply (**) with -λ yields

$$\left[\frac{d^2}{dt^2} + \alpha\frac{d}{dt} + \gamma\beta\right](p - p^*) = 0 \tag{22}$$

Equation (22) has a solution

$$p(t) = p^* + Al^{z_1 t} + Bl^{z_2 t} \tag{23}$$

with z_1 and z_2 are the roots from the equation

$$z^2 + \alpha z + \gamma\beta = 0 \tag{24}$$

Take $p(t)_{t=0} = p(0) \Rightarrow \left.\frac{dp}{dt}\right|_{t=0} = p'(t)\big|_{t=0} = p'(0)$ such that
with equation (23) if t=0, then

$$p(0) = p^* + Al^0 + Bl^0$$

$$p(0) = p^* + A + B$$

differensial equation (23) yields

$$\frac{dp}{dt} = p'(t) = z_1 A l^{z_1 t} + z_2 B l^{z_2 t} \text{ and}$$

$$A = \frac{z_2 \{p(0) - p^*\} - p'(0)}{z_2 - z_1} \tag{25}$$

From equation (20) if t = 0 is obtained

$$p'(0) + \alpha\left[p(0) - p^*\right] + \lambda\left[S(0) - S^*\right] = 0$$

Therefore

$$A = \frac{1}{z_2 - z_1}\left[\{z_2 + \alpha\}\{p(0) - p^*\} + \lambda\{S(0) - S^*\}\right] \tag{26}$$

In the same way, the value obtained

$$B = \frac{1}{z_1 - z_2}\left[\{z_1 + \alpha\}\{p(0) - p^*\} + \lambda\{S(0) - S^*\}\right] \tag{27}$$

Substitution from equation (23) into the expression D and φ produce equations to determine the flow from two variables.

Since all the coefficients in equation (24) is positive or remind that $b < 0$, so that $\alpha > 0$ and $\beta > 0$, this indicates that z_1 and z_2 have negative real numbers, so that the model is stable and all endogenous variables will tend to the equilibrium value of t $\rightarrow \infty$.
From equation (24) is obtained

$$z_1 \cdot z_2 = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\lambda\beta}}{2} \ ,$$

the solution is oscillatory (rotating) if $\alpha^2 - 4\lambda\beta < 0$, where α, λ, and β everything are positive, then obtained

$$\alpha = \gamma(d - b) - \lambda lb, \textit{ and } \beta = d - b. \text{ with } \alpha - 2\sqrt{\lambda\beta} < 0$$

then this inequality can be written as

$$\gamma(d - b) - \lambda lb - 2\sqrt{(d - b)\lambda} < 0 \tag{28}$$

considering $-b > 0$, then multiply this inequality with $\frac{l(-b)}{(d-b)}$ and substitution into equation (28), resulting in:

$$\frac{l^2(-b)^2\lambda}{d - b} - \frac{2l(-b)\sqrt{\lambda}}{\sqrt{d - b}} + l(-b)\gamma < 0 \tag{29}$$

which quadratic equation for $\sqrt{\lambda}$, the full root is then given

$$\left[\frac{l(-b)\sqrt{\lambda}}{\sqrt{d-b}}-1\right]^2 < 1-l(-b)\gamma \tag{30}$$

The combination from γ and λ that adjusted to the equation (30) and because it provides the solution oscillation can be shown in Figure 1.

With notes that, the influence of various factors can determine whether the convergent is averaged or oscillation. The weakness from adjustment price of adjustment excess demand price and supply in the market or for the deviation from the desire of action supplies is determined from the size of the parameters γ and λ and constants d,-b, and l.

On the position of $\gamma > l\,(-b)$, the adjustment is largely a result of an imbalance between supply and demand and its graph is a convergent evenly. The convergence is flat when $l\,(-b)^2\,\lambda > 4\,(d-b)$, but in this case because the price adjustment is very sensitive to deviations from the desire level of inventory when

$\gamma < l\,(-b)$ and $l^2\,(a-b)^2\,\lambda < 4\,(d-b)$ then the oscillation occurs, see Figure 1.

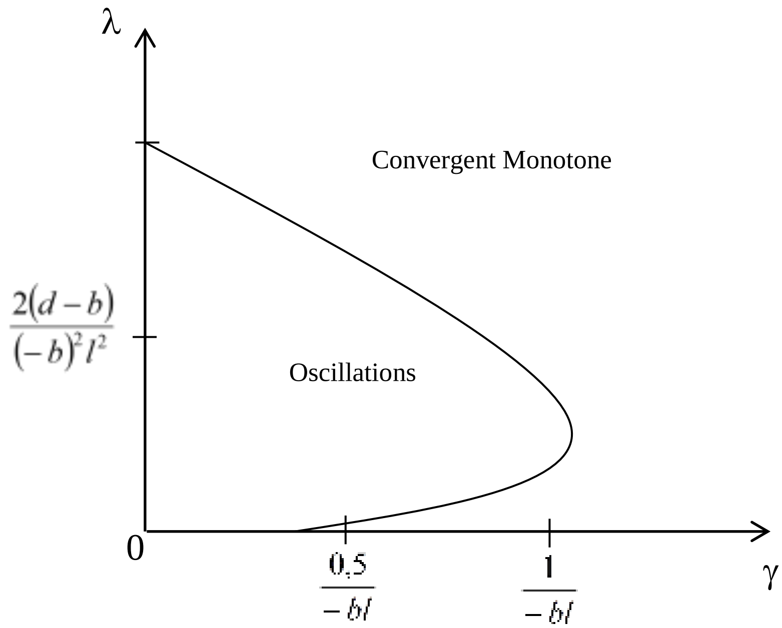

Figure 1. Graph of Convergent Monotone and Oscillations

4.2. Discrete Model Solution

Eliminate equation (11) and (12) into equation (13) which produces different order 2 equation, *i,e*:

$$bp_t - d\,(1+\rho)\,p_{t-1} + d\rho\,p_{t-2} = c - a \tag{31}$$

special solutions :

$$p_t = p_e \text{ with } p_e = \frac{c-a}{b-d} \tag{32}$$

y_e is the equilibrium value from the price.

Equation (31) multiplied with $\frac{1}{b}$

$$Pt - \frac{d(1+\rho)}{b} P_{t-1} + \frac{d\rho}{b} P_{t-2} = 0 \qquad (*)$$

Then the characteristic equation (*) is

$$\lambda^2 - \frac{d(1+\rho)}{b}\lambda + \frac{d\rho}{b} = 0 \qquad (33)$$

from equation (33), then the stable condition as a solution can be seen in Figure 2.

Furthermore if $1 - \frac{d(1+\rho)}{b} + \frac{d\rho}{b} = 1 - \frac{d}{b} > 0$ (34)

then $1 - \frac{d\rho}{b} > 0$ (35)

thus $1 + \frac{d(1+\rho)}{b} + \frac{d\rho}{b} = 1 + \frac{d(1+2\rho)}{b} > 0$ (36)

As usual assumed b < 0 and d > 0, thus the equation (34) automatically same with equation (35) and (36) then should be considered in analyzing this inequality, exactly when considere to be barrier on $d/{-b}$ and ρ. The Limit from equation (35) and (36) can be seen in Figure 2.

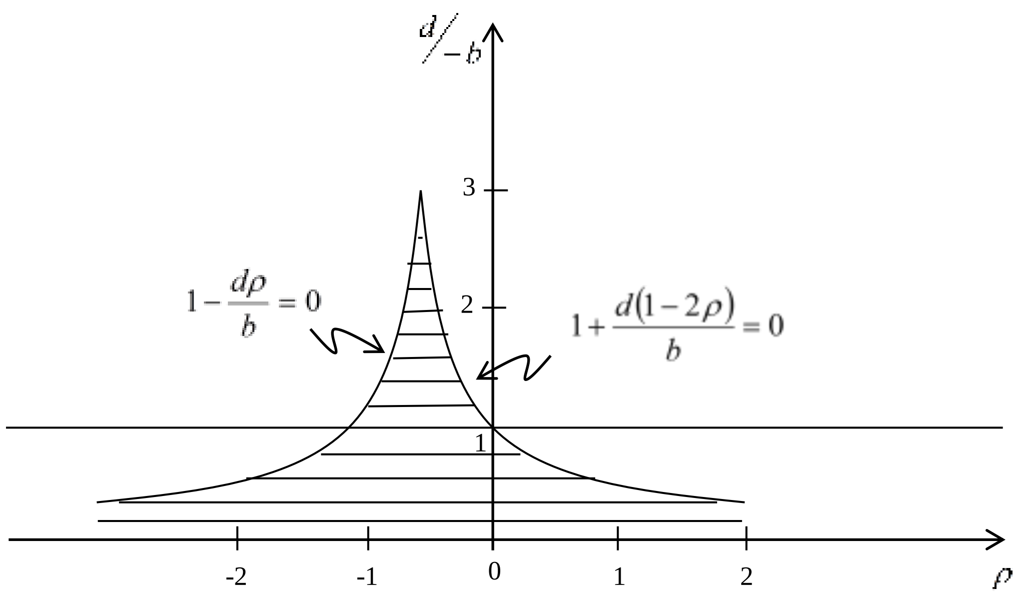

Figure 2. Graph On Current Stable Condition

Both this inequalities are adjusted for all points in the shadow areas. It should be noted that the only area of $\frac{d}{-b} > 0$, has been considered since assuming b <0 and d> 0, and does not include $\frac{d}{-b}$ a negative point, so that the intersection point is the limit from the inequality given by:

$$1 - \frac{d}{b}\rho = 1 + \frac{d}{b}(1 + 2\rho) = 0 \tag{37}$$

$$\rho = \frac{\frac{d}{b}}{-\frac{3d}{b}} \quad \text{and} \quad \frac{d}{-b} = \frac{-1}{-\frac{1}{3}}$$

which is the result $\rho = -\frac{1}{3}$ and $\frac{d}{-b} = 3$. At the time of $\rho < -\frac{1}{3}$ the inequality in equation (35) is more limited than the inequality in equation (36) and if $\rho > -\frac{1}{3}$ the equation (36) are preferred. Thus for the equation (35) and (36) if $\frac{d}{-b} > 0$ it is equivalent to

$$1 - \frac{d(1 + 2\rho)}{-b} > 0, \ \ for \ \rho > -\frac{1}{3} \tag{38}$$

and $$1 + \frac{d}{-b}\rho > 0, \ \ for \ \rho < -\frac{1}{3} \tag{39}$$

From equation (38) will produce

$$\frac{d}{-b} < \frac{1}{1 + 2\rho}, \quad for \ \rho > -\frac{1}{3} \tag{40}$$

From equation (39) can provide

$$1 - \frac{d}{-b}(-\rho) > 0, \quad for \ \ \rho < -\frac{1}{3} \tag{41}$$

then $$1 > \frac{d}{-b}(-\rho), \quad for \ \rho < -\frac{1}{3}$$

so that $$\frac{d}{-b} < \frac{1}{-\rho}, \ \ for \ \rho < -\frac{1}{3} \tag{42}$$

The stability from this model and the Cobweb model now can be compared. In Cobweb models, the stable condition are at $\frac{d}{-b} < 1$.

From Figure 2 or equation (40) and (42) it can be seen that the model flow is more stable from Cobweb models which provide that -1 < ρ < 0. Considering that the expected price in the interval is given by $\hat{p}_t = p_{t-1} + \rho(p_{t-1} - p_{t-2})$ and when ρ > 0,that the model is more unstable than Cobweb models so that the movement of the manufacture expectation price keep be conducted then net price in the market will be a previous net price plus an adjustment in leader changes, this obviously due to some instability. In addition, if $\rho < 0$, then the manufacturers will expect the price to be the opposite movement and expect that the opposite expectations are not too large, this expectation is an instability factor.

Furthermore, the discriminant from equation (33) is obtained

$$\Delta = \frac{d^2(1+\rho)^2}{b^2} - \frac{4d\rho}{b} \tag{43}$$

if ρ positive and $d > 0$, $b < 0$ then Δ is always positive. Thus the roots are real and apparent, therefore, the general solution from equation (31) can be written in the form:

$$p_t = A\lambda_1^{\,t} + B\lambda_2^{\,t} + \frac{c-a}{b-d} \tag{44}$$

with A and B are random constants and λ_1 and λ_2 are the roots from equation (33).

5. Conclusion

The sequence of the sign of the coefficient in the equation (33) in the discrete model is positive and negative, which is where the signs are following the rules of Descrates, that one root is positive and the other is negative. Similarly, the solution consists of a high position from not supposed oscillations and the movement is monotonik. When ρ is negative then Δ can take one of the signs. When Δ has been determined with order of signs and is set to any value that is provided from ρ, b and d, then the solutions of fixed form can be determined.

Furthermore, the continuous model is not be separated from the adjustment price of excess demand or excess supply in the market so that the parameters γ and λ with constants d, -b and l must be determined. This price adjustment is very sensitive once in a deviation from the level of inventory desire.

References

[1] Abdykappara. Ashimat,et.al. 2012. *Macroeconomics Analysis and Economic policy* **.** Macmillan Publishing Company, INC. New York.

[2] Ahmad Saleh, Mochamad Ridwan, Purmini, 2012 Analisis pertumbuhan Ekonomi Dan Kesenjangan Pembangunan Ekonomi Antar Region di Indonesia Tahun 2001-2010. *Jurnal Ekonomi Dan Perencanaan Pembangunan (JEPP)*, Vol 04, N0 03. Januari-Juni

[3] Cristoper.D. Piros CPA Jeral.E.Pinto Willey. 2013. *Economics For Invesment Dicision Market. Work book. Micro, Macro and International Economics.* Macmillan Publishing Company, INC. New York.

[4] David. L. Elements. 1983. *An Introduction to Mathematical Model In Economic Dynamics*. INC. Pub USA.

[5] Jeffrey. D. Sach, Felipe Lakrain. B. 1993. *Macroeconomics in The Global Economics.* Pretince-Hall, INC. New York.

[6] Richard. T. Froyen. 1993. *Macroeconomics, Theories and Polices.* Macmillan Publishing Company, INC. New York.

[7] Todaro, Michael, Stephen C Smith. 2008. *Pembangunan Ekonomi*. Edisi ke Sembilan, Jakarta, Erlangga.

[8] Wawan Wirahman, S. E. 1993. *Pengantar Makro Ekonomi*. Armico, Bandung.

[9] Yesi, Hendriani Supartoryo, Jen Tatuh, Ricky H. E.Sendouw. 2013. *The Economic Growth and The Regional Characteristics.* Buletin Ekonomi Moneter dan Perbangkan.